1.3 TRIGONOMETRIC FUNCTIONS

1. (a) $s = r\theta = (10)\left(\frac{4\pi}{5}\right) = 8\pi$ m (b) $s = r\theta = (10)(110°)\left(\frac{\pi}{180°}\right) = \frac{110\pi}{18} = \frac{55\pi}{9}$ m

2. $\theta = \frac{s}{r} = \frac{10\pi}{8} = \frac{5\pi}{4}$ radians and $\frac{5\pi}{4}\left(\frac{180°}{\pi}\right) = 225°$

3. $\theta = 80° \Rightarrow \theta = 80°\left(\frac{\pi}{180°}\right) = \frac{4\pi}{9} \Rightarrow s = (6)\left(\frac{4\pi}{9}\right) = 8.4$ in. (since the diameter = 12 in. $\Rightarrow$ radius = 6 in.)

4. d = 1 meter $\Rightarrow$ r = 50 cm $\Rightarrow \theta = \frac{s}{r} = \frac{30}{50} = 0.6$ rad or $0.6\left(\frac{180°}{\pi}\right) \approx 34°$

5.

θ	$-\pi$	$-\frac{2\pi}{3}$	0	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\sin\theta$	0	$-\frac{\sqrt{3}}{2}$	0	1	$\frac{1}{\sqrt{2}}$
$\cos\theta$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{\sqrt{2}}$
$\tan\theta$	0	$\sqrt{3}$	0	und.	-1
$\cot\theta$	und.	$\frac{1}{\sqrt{3}}$	und.	0	-1
$\sec\theta$	-1	-2	1	und.	$-\sqrt{2}$
$\csc\theta$	und.	$-\frac{2}{\sqrt{3}}$	und.	1	$\sqrt{2}$

6.

θ	$-\frac{3\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{5\pi}{6}$
$\sin\theta$	1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\cos\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$
$\tan\theta$	und.	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	1	$-\frac{1}{\sqrt{3}}$
$\cot\theta$	0	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	1	$-\sqrt{3}$
$\sec\theta$	und.	2	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$-\frac{2}{\sqrt{3}}$
$\csc\theta$	1	$-\frac{2}{\sqrt{3}}$	-2	$\sqrt{2}$	2

7. $\cos x = -\frac{4}{5}$, $\tan x = -\frac{3}{4}$

8. $\sin x = \frac{2}{\sqrt{5}}$, $\cos x = \frac{1}{\sqrt{5}}$

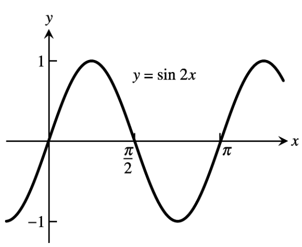
9. $\sin x = -\frac{\sqrt{8}}{3}$, $\tan x = -\sqrt{8}$

10. $\sin x = \frac{12}{13}$, $\tan x = -\frac{12}{5}$

11. $\sin x = -\frac{1}{\sqrt{5}}$, $\cos x = -\frac{2}{\sqrt{5}}$

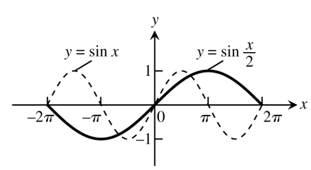
12. $\cos x = -\frac{\sqrt{3}}{2}$, $\tan x = \frac{1}{\sqrt{3}}$

13.



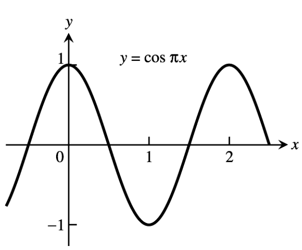
period $= \pi$

14.



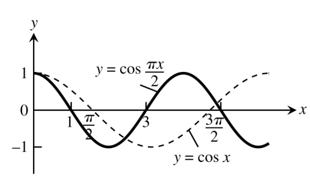
period $= 4\pi$

15.



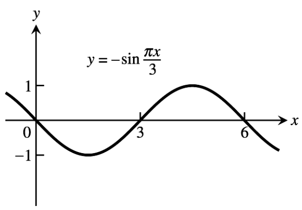
period $= 2$

16.



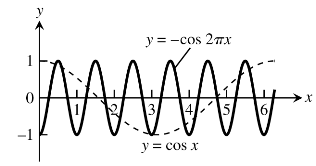
period $= 4$

17.



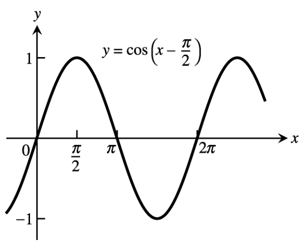
period $= 6$

18.



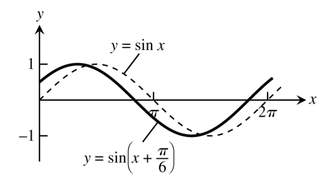
period $= 1$

19.



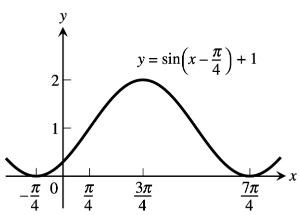
period $= 2\pi$

20.



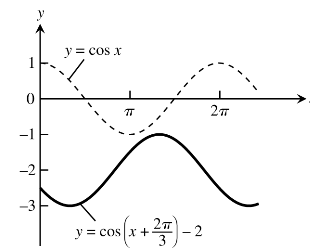
period $= 2\pi$

21.



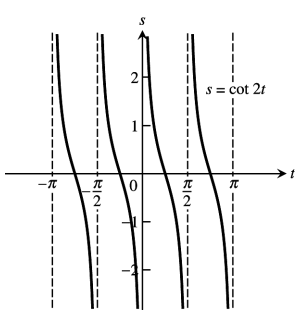
period $= 2\pi$

22.

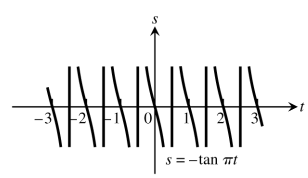


period $= 2\pi$

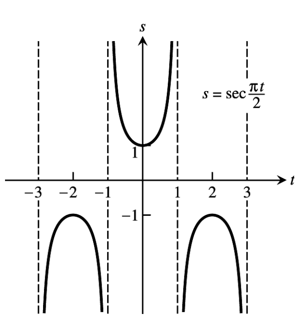
23. period $= \frac{\pi}{2}$, symmetric about the origin



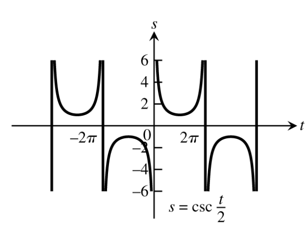
24. period $= 1$, symmetric about the origin



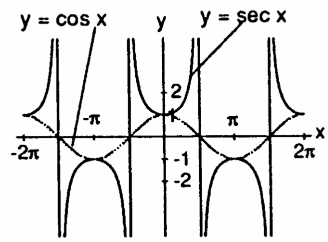
25. period $= 4$, symmetric about the s-axis



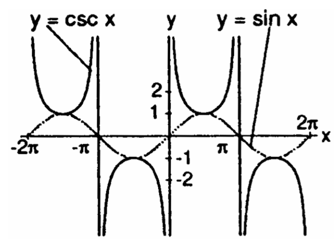
26. period $= 4\pi$, symmetric about the origin



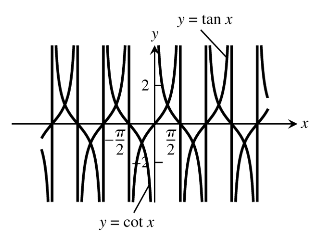
27. (a) Cos x and sec x are positive for x in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; and cos x and sec x are negative for x in the intervals $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right)$ and $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$. Sec x is undefined when cos x is 0. The range of sec x is $(-\infty, -1] \cup [1, \infty)$; the range of cos x is $[-1, 1]$.



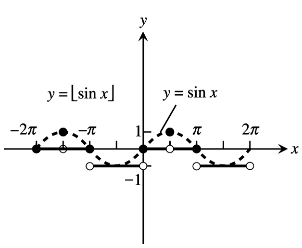
(b) Sin x and csc x are positive for x in the intervals $\left(-\frac{3\pi}{2}, -\pi\right)$ and $(0, \pi)$; and sin x and csc x are negative for x in the intervals $(-\pi, 0)$ and $\left(\pi, \frac{3\pi}{2}\right)$. Csc x is undefined when sin x is 0. The range of csc x is $(-\infty, -1] \cup [1, \infty)$; the range of sin x is $[-1, 1]$.



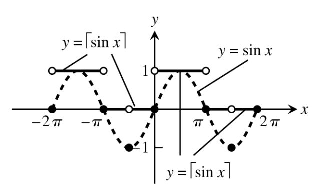
28. Since $\cot x = \frac{1}{\tan x}$, cot x is undefined when $\tan x = 0$ and is zero when tan x is undefined. As tan x approaches zero through positive values, cot x approaches infinity. Also, cot x approaches negative infinity as tan x approaches zero through negative values.



29. D: $-\infty < x < \infty$; R: $y = -1, 0, 1$



30. D: $-\infty < x < \infty$; R: $y = -1, 0, 1$



31. $\cos\left(x - \frac{\pi}{2}\right) = \cos x \cos\left(-\frac{\pi}{2}\right) - \sin x \sin\left(-\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(-1) = \sin x$

32. $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\left(\frac{\pi}{2}\right) - \sin x \sin\left(\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(1) = -\sin x$

33. $\sin\left(x + \frac{\pi}{2}\right) = \sin x \cos\left(\frac{\pi}{2}\right) + \cos x \sin\left(\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(1) = \cos x$

34. $\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\left(-\frac{\pi}{2}\right) + \cos x \sin\left(-\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(-1) = -\cos x$

35. $\cos(A - B) = \cos(A + (-B)) = \cos A \cos(-B) - \sin A \sin(-B) = \cos A \cos B - \sin A(-\sin B)$
$= \cos A \cos B + \sin A \sin B$

36. $\sin(A - B) = \sin(A + (-B)) = \sin A \cos(-B) + \cos A \sin(-B) = \sin A \cos B + \cos A(-\sin B)$
$= \sin A \cos B - \cos A \sin B$

37. If $B = A$, $A - B = 0 \Rightarrow \cos(A - B) = \cos 0 = 1$. Also $\cos(A - B) = \cos(A - A) = \cos A \cos A + \sin A \sin A$ $= \cos^2 A + \sin^2 A$. Therefore, $\cos^2 A + \sin^2 A = 1$.

38. If $B = 2\pi$, then $\cos(A + 2\pi) = \cos A \cos 2\pi - \sin A \sin 2\pi = (\cos A)(1) - (\sin A)(0) = \cos A$ and $\sin(A + 2\pi) = \sin A \cos 2\pi + \cos A \sin 2\pi = (\sin A)(1) + (\cos A)(0) = \sin A$. The result agrees with the fact that the cosine and sine functions have period 2π.

39. $\cos(\pi + x) = \cos\pi \cos x - \sin\pi \sin x = (-1)(\cos x) - (0)(\sin x) = -\cos x$

40. $\sin(2\pi - x) = \sin 2\pi \cos(-x) + \cos(2\pi)\sin(-x) = (0)(\cos(-x)) + (1)(\sin(-x)) = -\sin x$

41. $\sin\left(\frac{3\pi}{2} - x\right) = \sin\left(\frac{3\pi}{2}\right)\cos(-x) + \cos\left(\frac{3\pi}{2}\right)\sin(-x) = (-1)(\cos x) + (0)(\sin(-x)) = -\cos x$

42. $\cos\left(\frac{3\pi}{2} + x\right) = \cos\left(\frac{3\pi}{2}\right)\cos x - \sin\left(\frac{3\pi}{2}\right)\sin x = (0)(\cos x) - (-1)(\sin x) = \sin x$

43. $\sin\frac{7\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$

44. $\cos\frac{11\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{\pi}{4}\sin\frac{2\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2}+\sqrt{6}}{4}$

45. $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\left(-\frac{\pi}{4}\right) - \sin\frac{\pi}{3}\sin\left(-\frac{\pi}{4}\right) = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$

46. $\sin\frac{5\pi}{12} = \sin\left(\frac{2\pi}{3} - \frac{\pi}{4}\right) = \sin\left(\frac{2\pi}{3}\right)\cos\left(-\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{3}\right)\sin\left(-\frac{\pi}{4}\right) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$

47. $\cos^2\frac{\pi}{8} = \frac{1+\cos\left(\frac{2\pi}{8}\right)}{2} = \frac{1+\frac{\sqrt{2}}{2}}{2} = \frac{2+\sqrt{2}}{4}$

48. $\cos^2\frac{5\pi}{12} = \frac{1+\cos\left(\frac{10\pi}{12}\right)}{2} = \frac{1+\left(-\frac{\sqrt{3}}{2}\right)}{2} = \frac{2-\sqrt{3}}{4}$

49. $\sin^2\frac{\pi}{12} = \frac{1-\cos\left(\frac{2\pi}{12}\right)}{2} = \frac{1-\frac{\sqrt{3}}{2}}{2} = \frac{2-\sqrt{3}}{4}$

50. $\sin^2\frac{3\pi}{8} = \frac{1-\cos\left(\frac{6\pi}{8}\right)}{2} = \frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{2} = \frac{2+\sqrt{2}}{4}$

51. $\sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

52. $\sin^2\theta = \cos^2\theta \Rightarrow \frac{\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} \Rightarrow \tan^2\theta = 1 \Rightarrow \tan\theta = \pm 1 \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

53. $\sin 2\theta - \cos\theta = 0 \Rightarrow 2\sin\theta\cos\theta - \cos\theta = 0 \Rightarrow \cos\theta(2\sin\theta - 1) = 0 \Rightarrow \cos\theta = 0$ or $2\sin\theta - 1 = 0 \Rightarrow \cos\theta = 0$ or $\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$, or $\theta = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

54. $\cos 2\theta + \cos\theta = 0 \Rightarrow 2\cos^2\theta - 1 + \cos\theta = 0 \Rightarrow 2\cos^2\theta + \cos\theta - 1 = 0 \Rightarrow (\cos\theta + 1)(2\cos\theta - 1) = 0$ $\Rightarrow \cos\theta + 1 = 0$ or $2\cos\theta - 1 = 0 \Rightarrow \cos\theta = -1$ or $\cos\theta = \frac{1}{2} \Rightarrow \theta = \pi$ or $\theta = \frac{\pi}{3}, \frac{5\pi}{3} \Rightarrow \theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

55. $\tan(A + B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A\cos B + \cos A\cos B}{\cos A\cos B - \sin A\sin B} = \frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

56. $\tan(A - B) = \frac{\sin(A-B)}{\cos(A-B)} = \frac{\sin A\cos B - \cos A\cos B}{\cos A\cos B + \sin A\sin B} = \frac{\frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} + \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

57. According to the figure in the text, we have the following: By the law of cosines, $c^2 = a^2 + b^2 - 2ab\cos\theta$ $= 1^2 + 1^2 - 2\cos(A - B) = 2 - 2\cos(A - B)$. By distance formula, $c^2 = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$ $= \cos^2 A - 2\cos A\cos B + \cos^2 B + \sin^2 A - 2\sin A\sin B + \sin^2 B = 2 - 2(\cos A\cos B + \sin A\sin B)$. Thus $c^2 = 2 - 2\cos(A - B) = 2 - 2(\cos A\cos B + \sin A\sin B) \Rightarrow \cos(A - B) = \cos A\cos B + \sin A\sin B$.

58. (a) $\cos(A-B) = \cos A \cos B \ + \sin A \sin B$
$\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$ and $\cos\theta = \sin\left(\frac{\pi}{2}-\theta\right)$
Let $\theta = A+B$
$\sin(A+B) = \cos\left[\frac{\pi}{2}-(A+B)\right] = \cos\left[\left(\frac{\pi}{2}-A\right)-B\right] = \cos\left(\frac{\pi}{2}-A\right)\cos B \ + \sin\left(\frac{\pi}{2}-A\right)\sin B$
$= \sin A \cos \ B + \cos A \sin B$
(b) $\cos(A-B) = \cos A \cos B \ + \sin A \sin B$
$\cos(A-(-B)) = \cos A \cos(-B) \ + \sin A \sin(-B)$
$\Rightarrow \cos(A+B) = \cos A \cos(-B) \ + \sin A \sin(-B) = \cos A \cos B \ + \sin A(-\sin B)$
$= \cos A \cos B \ - \sin A \sin B$
Because the cosine function is even and the sine functions is odd.

59. $c^2 = a^2 + b^2 - 2ab\cos C = 2^2 + 3^2 - 2(2)(3)\cos(60°) = 4 + 9 - 12\cos(60°) = 13 - 12\left(\frac{1}{2}\right) = 7.$
Thus, $c = \sqrt{7} \approx 2.65.$

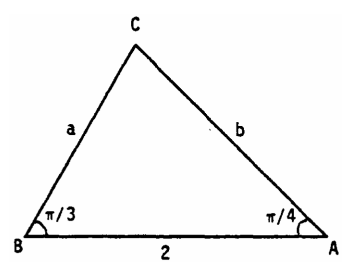
60. $c^2 = a^2 + b^2 - 2ab\cos C = 2^2 + 3^2 - 2(2)(3)\cos(40°) = 13 - 12\cos(40°)$. Thus, $c = \sqrt{13 - 12\cos 40°} \approx 1.951.$

61. From the figures in the text, we see that $\sin B = \frac{h}{c}$. If C is an acute angle, then $\sin C = \frac{h}{b}$. On the other hand, if C is obtuse (as in the figure on the right), then $\sin C = \sin(\pi - C) = \frac{h}{b}$. Thus, in either case, $h = b\sin C = c\sin B \ \Rightarrow \ ah = ab\sin C = ac\sin B.$

By the law of cosines, $\cos C = \frac{a^2+b^2-c^2}{2ab}$ and $\cos B = \frac{a^2+c^2-b^2}{2ac}$. Moreover, since the sum of the interior angles of a triangle is π, we have $\sin A = \sin(\pi-(B+C)) = \sin(B+C) = \sin B\cos C + \cos B\sin C$
$= \left(\frac{h}{c}\right)\left[\frac{a^2+b^2-c^2}{2ab}\right] + \left[\frac{a^2+c^2-b^2}{2ac}\right]\left(\frac{h}{b}\right) = \left(\frac{h}{2abc}\right)(2a^2+b^2-c^2+c^2-b^2) = \frac{ah}{bc} \ \Rightarrow \ ah = bc\sin A.$
Combining our results we have $ah = ab\sin C$, $ah = ac\sin B$, and $ah = bc\sin A$. Dividing by abc gives
$\frac{h}{bc} = \underbrace{\frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}}_{\text{law of sines}}.$

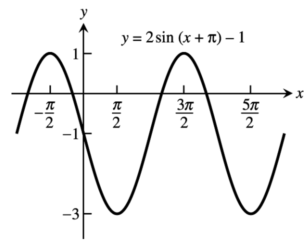
62. By the law of sines, $\frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sqrt{3}/2}{c}$. By Exercise 61 we know that $c = \sqrt{7}$. Thus $\sin B = \frac{3\sqrt{3}}{2\sqrt{7}} \simeq 0.982.$

63. From the figure at the right and the law of cosines,
$b^2 = a^2 + 2^2 - 2(2a)\cos B$
$= a^2 + 4 - 4a\left(\frac{1}{2}\right) = a^2 - 2a + 4.$
Applying the law of sines to the figure, $\frac{\sin A}{a} = \frac{\sin B}{b}$
$\Rightarrow \frac{\sqrt{2}/2}{a} = \frac{\sqrt{3}/2}{b} \ \Rightarrow \ b = \sqrt{\frac{3}{2}}\,a$. Thus, combining results,
$a^2 - 2a + 4 = b^2 = \frac{3}{2}a^2 \ \Rightarrow \ 0 = \frac{1}{2}a^2 + 2a - 4$
$\Rightarrow \ 0 = a^2 + 4a - 8$. From the quadratic formula and the fact that $a > 0$, we have
$a = \frac{-4+\sqrt{4^2-4(1)(-8)}}{2} = \frac{4\sqrt{3}-4}{2} \simeq 1.464.$

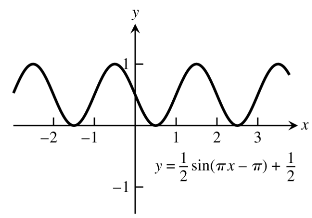


64. (a) The graphs of $y = \sin x$ and $y = x$ nearly coincide when x is near the origin (when the calculator is in radians mode).
(b) In degree mode, when x is near zero degrees the sine of x is much closer to zero than x itself. The curves look like intersecting straight lines near the origin when the calculator is in degree mode.

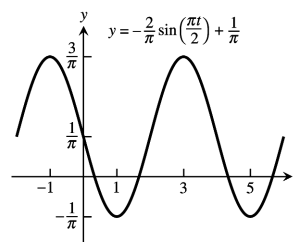
65. $A = 2, B = 2\pi, C = -\pi, D = -1$



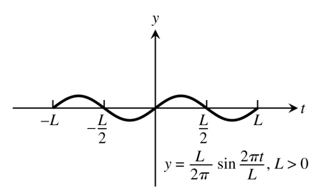
66. $A = \frac{1}{2}, B = 2, C = 1, D = \frac{1}{2}$



67. $A = -\frac{2}{\pi}, B = 4, C = 0, D = \frac{1}{\pi}$



68. $A = \frac{L}{2\pi}, B = L, C = 0, D = 0$



69-72. Example CAS commands:

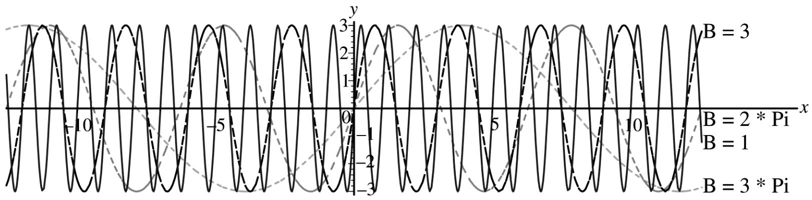
<u>Maple</u>

```
f := x -> A*sin((2*Pi/B)*(x-C))+D1;
A:=3; C:=0; D1:=0;
f_list := [seq( f(x), B=[1,3,2*Pi,5*Pi] )];
plot( f_list, x=-4*Pi..4*Pi, scaling=constrained,
      color=[red,blue,green,cyan], linestyle=[1,3,4,7],
      legend=["B=1","B=3","B=2*Pi","B=3*Pi"],
      title="#69 (Section 1.3)" );
```

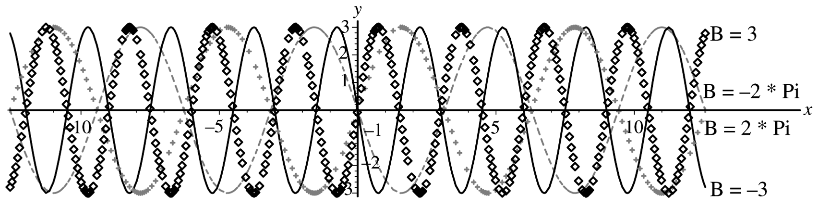
<u>Mathematica</u>

```
Clear[a, b, c, d, f, x]
f[x_]:=a  Sin[2π/b (x − c)] + d
Plot[f[x]/.{a → 3, b → 1, c → 0, d → 0}, {x, −4π, 4π }]
```

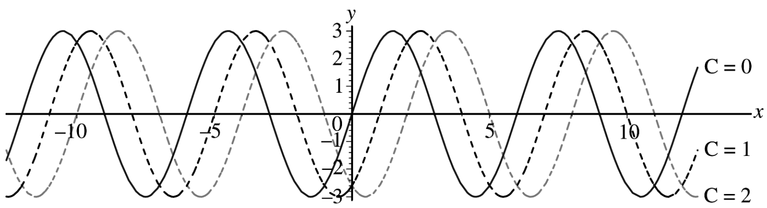
69. (a) The graph stretches horizontally.



(b) The period remains the same: period $= |\,B\,|$. The graph has a horizontal shift of $\frac{1}{2}$ period.



70. (a) The graph is shifted right C units.

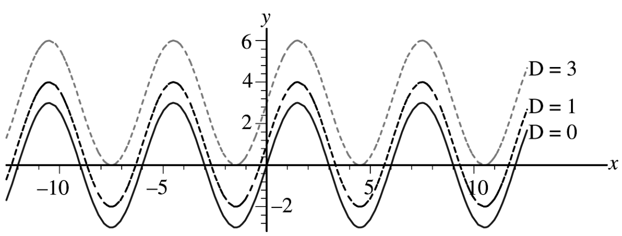


(b) The graph is shifted left C units.

(c) A shift of $\pm$ one period will produce no apparent shift. $|\,C\,| = 6$

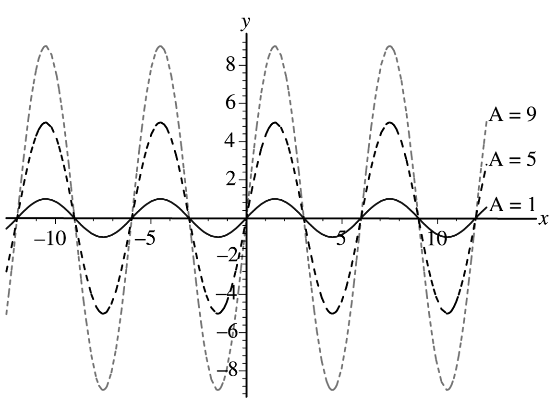
71. (a) The graph shifts upwards $|\,D\,|$units for $D > 0$

(b) The graph shifts down $|\,D\,|$units for $D < 0$.



72. (a) The graph stretches $|\,A\,|$ units.

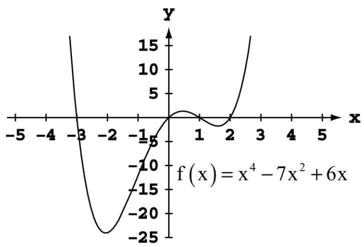
(b) For $A < 0$, the graph is inverted.



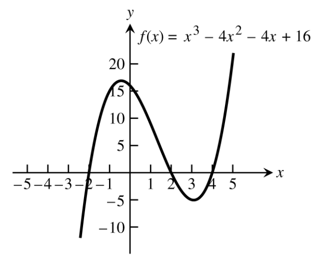
1.4 GRAPHING WITH CALCULATORS AND COMPUTERS

1-4. The most appropriate viewing window displays the maxima, minima, intercepts, and end behavior of the graphs and has little unused space.

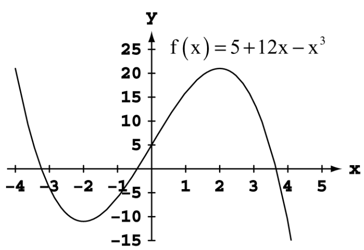
1. d.



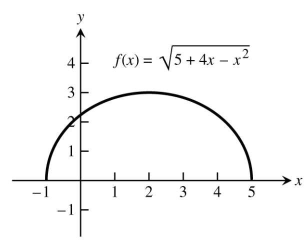
2. c.



3. d.

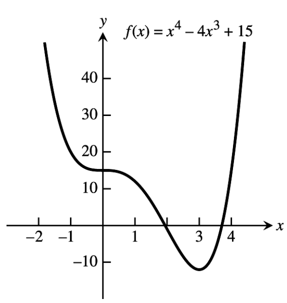


4. b.

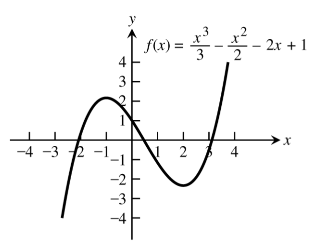


5-30. For any display there are many appropriate display widows. The graphs given as answers in Exercises 5−30 are not unique in appearance.

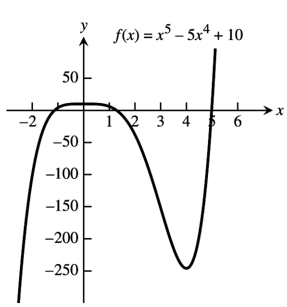
5. $[-2, 5]$ by $[-15, 40]$



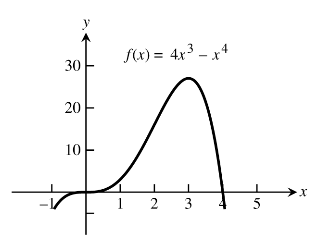
6. $[-4, 4]$ by $[-4, 4]$



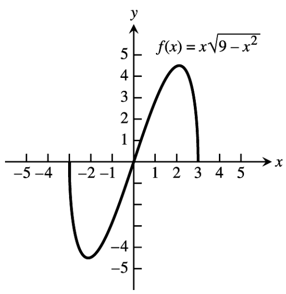
7. $[-2, 6]$ by $[-250, 50]$



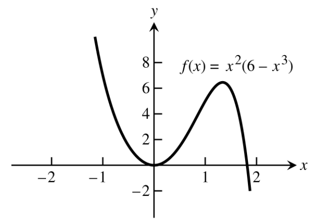
8. $[-1, 5]$ by $[-5, 30]$



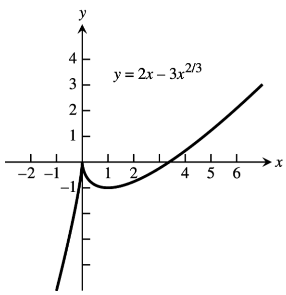
9. $[-4, 4]$ by $[-5, 5]$



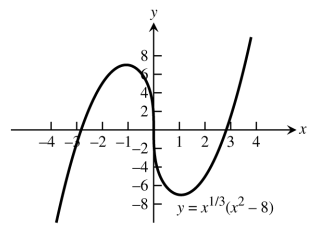
10. $[-2, 2]$ by $[-2, 8]$



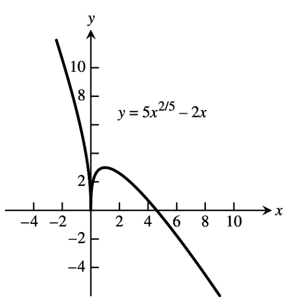
11. $[-2, 6]$ by $[-5, 4]$



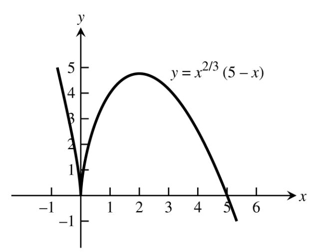
12. $[-4, 4]$ by $[-8, 8]$



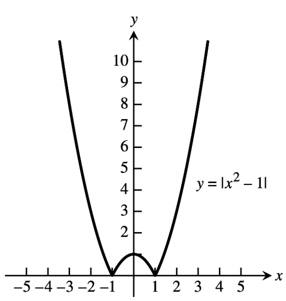
13. $[-1, 6]$ by $[-1, 4]$



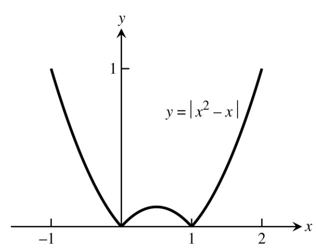
14. $[-1, 6]$ by $[-1, 5]$



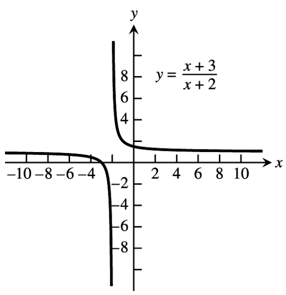
15. $[-3, 3]$ by $[0, 10]$



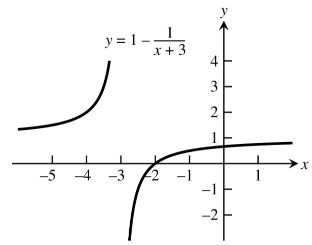
16. $[-1, 2]$ by $[0, 1]$



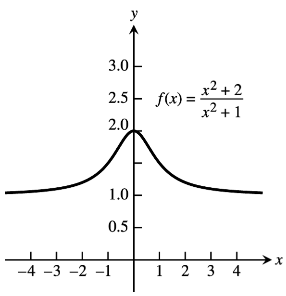
17. $[-5, 1]$ by $[-5, 5]$



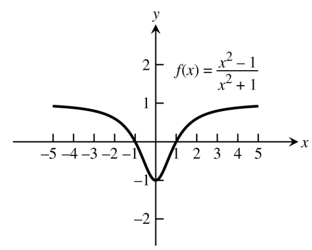
18. $[-5, 1]$ by $[-2, 4]$



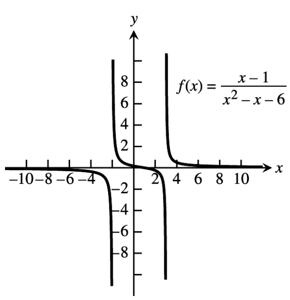
19. $[-4, 4]$ by $[0, 3]$



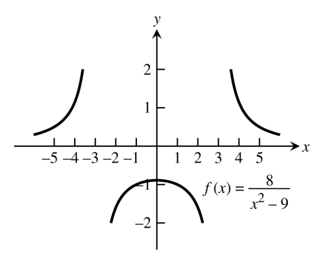
20. $[-5, 5]$ by $[-2, 2]$



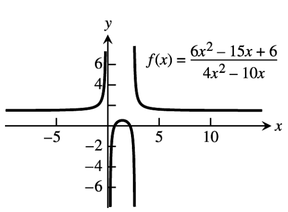
21. $[-10, 10]$ by $[-6, 6]$



22. $[-5, 5]$ by $[-2, 2]$



23. $[-6, 10]$ by $[-6, 6]$



24. $[-3, 5]$ by $[-2, 10]$